

# Direct reactions – generic features

- 1) Reactions in which there is a minimal rearrangement, or excitation involving a <u>very small number</u> of active (*effective*) degrees of freedom of the projectile and/or target: single-particle (sp) or collective inelastic excitation, sp or cluster transfers or knockout '<u>fast</u>'
- 2) Reaction energies are such that <u>average</u>, <u>effective</u> (complex) interactions can be used between the reacting constituents regions of high level density
- 3) Because of complex effective interactions, and short mean free paths, reactions are localised / dominated by interactions in the nuclear surfaces and by hence by peripheral and grazing collisions.



#### Direct reactions – literature

- 1. N. Austern, Direct Nuclear Reaction Theories, Wiley (NY), 1970
- 2. G.R. Satchler, *Direct Nuclear Reactions*, Clarendon OUP, 1983
- F.S. Levin and H. Feshbach, Reaction Dynamics, Gordon and Breach, 1973
- 4. H. Feshbach, *Theoretical Nuclear Physics: Nuclear Reactions*, Wiley, N.Y. 1992
- 5. Various authors, *Nuclear Spectroscopy and Reactions*, Vols. A-D, Ed. J. Cerny, Academic Press, 1974

#### **Topical Reviews**

- 6. J.Al-Khalili and J.A. Tostevin, Few-Body Models of Nuclear Reactions, Chapter 3.4, in Scattering, Academic Press, 2002
- 7. P.G. Hansen and J.A. Tostevin, *Direct Reactions of Exotic Nuclei*, ARNPS **53** (2003) 219-261
- 8. J.Al-Khalili and F.M. Nunes, J. Phys. G: Nucl. Part. Phys. **29** (2003) R89-R132.

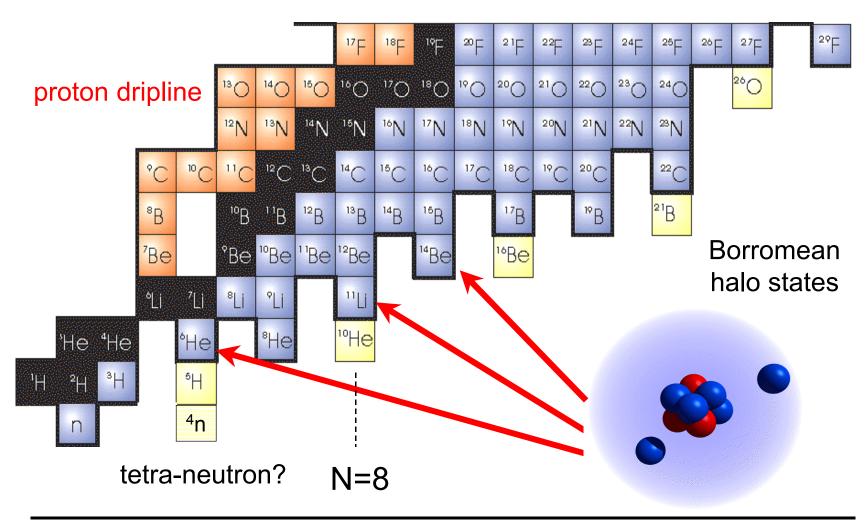


# Direct reactions with light-ions: questions were?

- 1) How important is it to take account of the <u>loosely bound</u> nature of the deuteron/triton/<sup>3</sup>He and three-body breakup channels in direct reactions?
- 2) How accurate are first-order (BA, DWBA) approaches, and the <u>spectroscopic information</u> (spectroscopic factors B(E2)'s, deformations and angular momentum assignments) deduced, as a test of structure models?
- 3) How can one treat 'practically' few- and many-body nuclear reactions in a <u>non-perturbative</u> (non-BA / all order) manner?
- 4) How does one deal with the sensitivity of direct reaction calculations to the assumed effective interactions?



# The new deuterons - the driplines in light nuclei





#### Direct reactions – many experimental advances

- Single-nucleon transfer reactions, HRIBF, ISOLDE,
   GANIL, ANL, Texas A&M, ... + new detector arrays
- Coulomb excitation / break-up: RIKEN, MSU, GSI, ...
- Elastic and inelastic scattering, many groups, ...
- Break-up reactions, ND, MSU, RIKEN, GSI ...
- One- and two-nucleon knockout, MSU, GANIL, GSI, RIKEN

Much direct reaction theory 'of old' can be carried over to exotics arena – but weakly bound exotic systems are a new challenge (non-perturbative, non-DWBA)

Many important lessons were learned – avoid reinvention



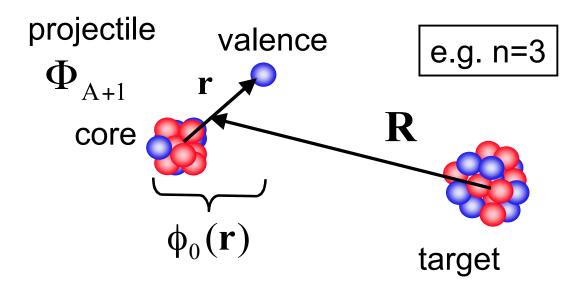
# Essential points/questions with exotic beams:

- 1) It is vital to take into account the <u>loosely bound</u> nature of exotic nuclei and their break-up channels in calculations of reaction observables
- 2) How accurate is <u>spectroscopic information</u> (spectroscopic factors and angular momentum assignments) deduced from approximate few-body models as a test of sophisticated structure models?
- 3) How can one treat 'practically' few- and many-body nuclear reactions in a non-perturbative manner?
- 4) How can we best choose the assumed effective interactions between reacting constituents?



#### Few-body models of nuclear reactions

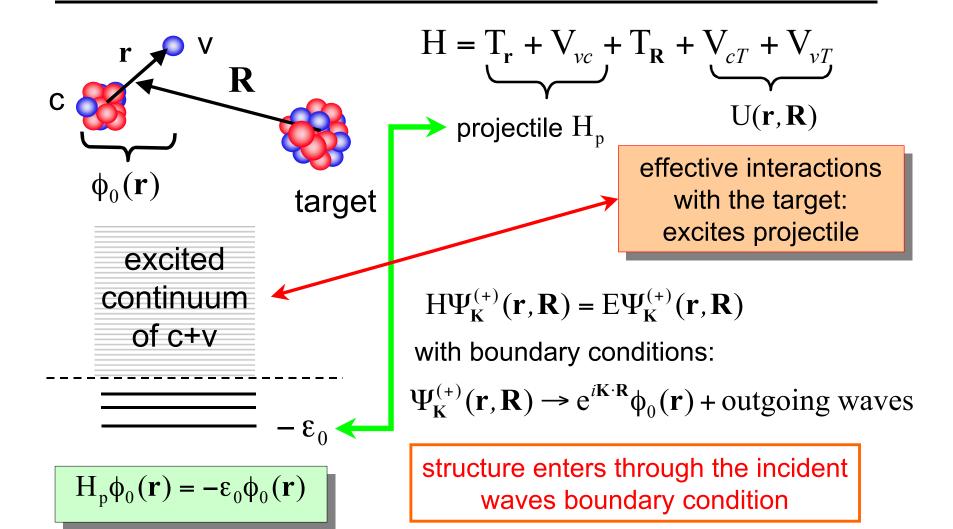
There are no practical many-body reaction theories - we construct model 'effective' few-body models (n=2,3,4 ...)



Construct an <u>effective</u> Hamiltonian H and solve as best we can the Schrödinger equation:  $H\Psi = E\Psi$ 



# Few-body reaction theory - definitions - notation



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#### Few-body models - effective interactions

$$H = T_{r} + V_{vc} + T_{R} + V_{cT} + V_{vT}$$
 effective (complex) interactions of c and v individually with target (nuclear + Coulomb potentials)

- (a) From experiment: potentials fitted to available data for c+T or v+T scattering at the appropriate energy per nucleon
- (b) From theory: multiple scattering or folding models, for example

$$V_{cT}(R) = \int d\mathbf{r}_1 \int d\mathbf{r}_2 \, \rho_c(\mathbf{r}_1) \, \rho_T(\mathbf{r}_2) \, \mathbf{t}_{NN}(\mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1)$$
core and target densities

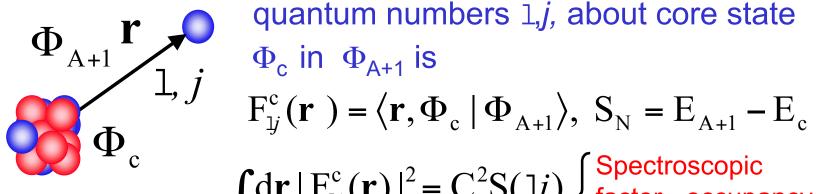
nucleon-nucleon t-matrix or effective NN interaction



# Structure information – overlap integrals

Nucleon removal from  $\Phi_{A+1}$  will leave mass A residue in the ground or an excited state - even in extreme sp model

More generally:



amplitude for finding nucleon with sp quantum numbers 1,j, about core state

$$\Phi_{\rm c}$$
 in  $\Phi_{\rm A+1}$  is

$$F_{jj}^{c}(\mathbf{r}) = \langle \mathbf{r}, \Phi_{c} | \Phi_{A+1} \rangle, S_{N} = E_{A+1} - E_{c}$$

$$\int d\mathbf{r} |F_{jj}^{c}(\mathbf{r})|^{2} = C^{2}S(j) \begin{cases} \text{Spectroscopic} \\ \text{factor - occupancy} \\ \text{of the state} \end{cases}$$

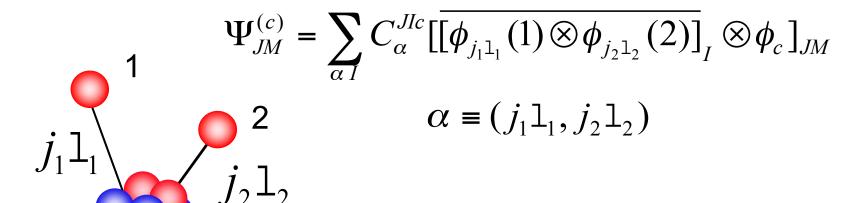
Usual to write

$$F_{1j}^{c}(\mathbf{r}) = \sqrt{C^{2}S(1j)} \phi_{0}(\mathbf{r}); \quad \int d\mathbf{r} |\phi_{0}(\mathbf{r})|^{2} = 1$$

with  $\phi_0(\mathbf{r})$  calculated in a potential model (Woods-Saxon)



#### Structure information – two nucleon overlaps



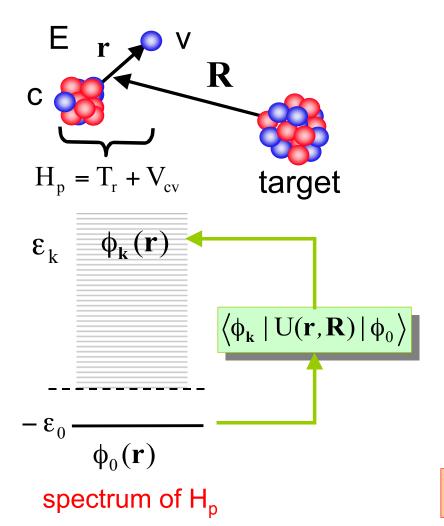
There is now no factorisation – but a coherent superposition

 $^{28}Mg \rightarrow ^{26}Ne(0^{+})$ 

$$C(2s_{1/2})^2 = -0.305$$
  
 $C(1d_{3/2})^2 = -0.301$   
 $C(1d_{5/2})^2 = -1.05$ 



# Energetics of few-body composite systems



$$H = H_p + T_R + U(r, R)$$

The tidal forces  $U(\mathbf{r}, \mathbf{R}) = V_{rT} + V_{rT}$ between c and v and the target cause excitation of the projectile to excited states of c+v and to the continuum states

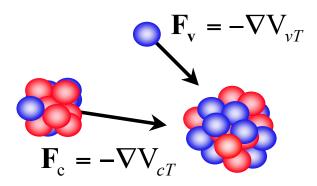
$$H_{p}\phi_{k}(\mathbf{r}) = \varepsilon_{k}\phi_{k}(\mathbf{r})$$

Which  $\phi_{k}(\mathbf{r})$  are excited?

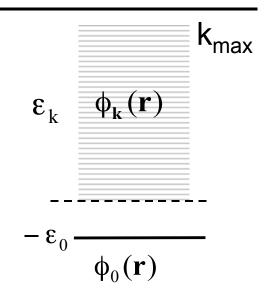


#### Continuum excitations and interactions

A major simplification to the reaction dynamics is possible if  $\epsilon_k \ll E$ 



Those states excited (to k<sub>max</sub>) are dictated by the geometry of the interactions



Nuclear forces, sharp surfaces, large  $\mathbf{F}$ , larger  $\epsilon_k$ , universally, given surface diffuseness of nuclear potentials  $\epsilon_k \le 20 \text{ MeV}$ 

Coulomb forces, slow spatial changes, small  $\mathbf{F}$ , typically  $\epsilon_k \le 4$  MeV (e.g. Nakamura et al, PRL **83** (1998) 1112)

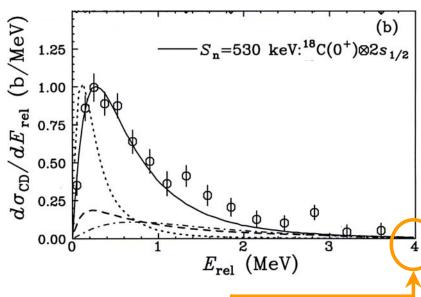
In both cases, for the energies if RI beams from fragmentation facilities (50-100 MeV per nucleon), typical  $\langle H_p \rangle << E$ 

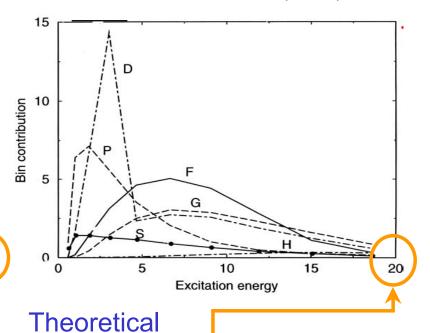


#### Break-up continua from nuclear and Coulomb

T. Nakamura et al, PRL 83 (1998) 1112

J.A. Tostevin et al, PRC **66** (2002) 024607





#### **Experimental**

 $^{19}\text{C+Pb} \rightarrow ^{18}\text{C+n+X}$ 

E= 67A MeV = 1.33 GeV

**Coulomb** dominated

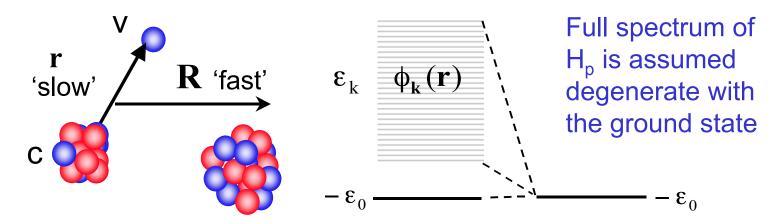
<sup>11</sup>Be + <sup>9</sup>Be → <sup>10</sup>Be+n+X

E= 60A MeV = 660 MeV

**Nuclear** dominated



#### Adiabatic model for few-body projectiles



Freeze internal co-ordinate r then scatter c+v from target and compute  $f(\theta, r)$  for all required <u>fixed</u> values of r

Physical amplitude for breakup to state  $\phi_k(\mathbf{r})$  is then,

$$f_k(\theta) = \langle \phi_k | f(\theta, r) | \phi_0 \rangle_r$$

Achieved by replacing  $H_p \rightarrow -\epsilon_0$  in Schrödinger equation



# Adiabatic approximation - time perspective

The time-dependent equation is

$$H\Psi(\mathbf{r}, \mathbf{R}, t) = ih \frac{\partial \Psi}{\partial t}$$

and can be written

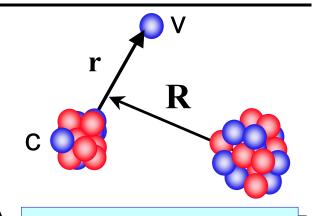
$$\Psi(\mathbf{r}, \mathbf{R}, t) = \Lambda \Phi(\mathbf{r}(t), \mathbf{R}), \quad \mathbf{r}(t) = \Lambda^{+} \mathbf{r} \Lambda$$

$$\Lambda = \exp\{-i(H_p + \epsilon_0)t/h\}$$
 and where

$$[T_R + U(\mathbf{r}(t), \mathbf{R}) - \varepsilon_0]\Phi(\mathbf{r}(t), \mathbf{R}) = i\hbar \frac{\partial \Phi}{\partial t}$$

Adiabatic equation

$$[T_R + U(\mathbf{r}, \mathbf{R})]\Phi(\mathbf{r}, \mathbf{R}) = (E + \varepsilon_0)\Phi(\mathbf{r}, \mathbf{R})$$



Adiabatic step assumes

$$r(t) \approx r(0) = r = fixed$$
  
or  $\Lambda = 1$  for the  
collision time  $t_{coll}$ 

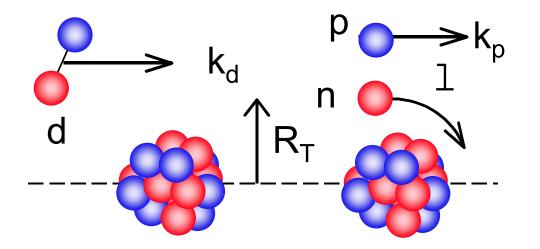
requires

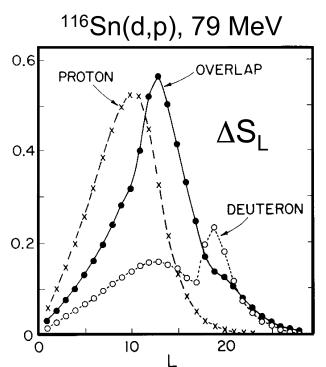
$$(H_p + \varepsilon_0)t_{coll}/h << 1$$



# Transfer reactions: matching conditions

$$T_{dp} = \left\langle \chi_p^{(-)}(\mathbf{r}_p) \phi_n(\mathbf{r}_n) \middle| V_{np} \middle| \Psi_K^{(+)}(\mathbf{r}, \mathbf{R}) \right\rangle$$



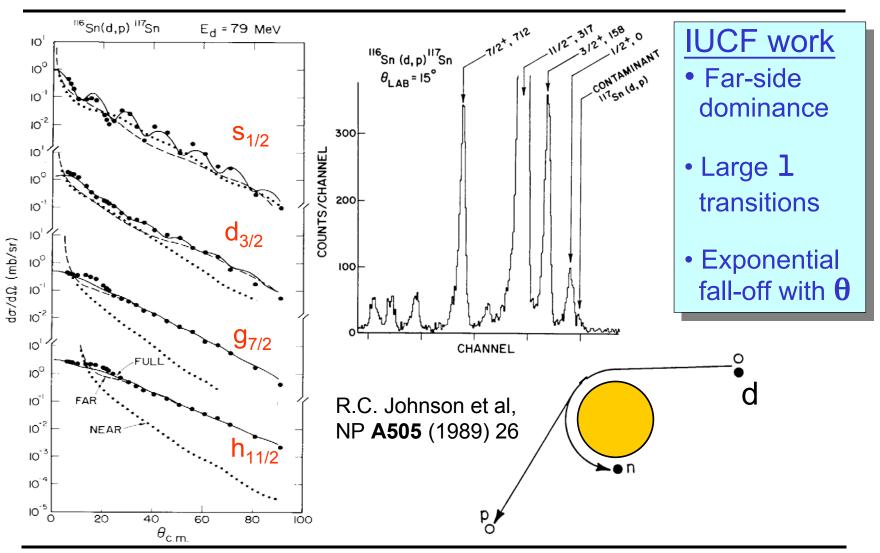


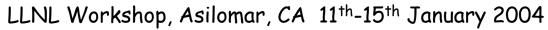
angular momentum matching  $1 \approx |L_d - L_{p|} \sim |k_d - k_p| \times R_T$ linear momentum mismatch  $\Delta k \approx |k_d/2 - k_p|$ 

 $E_p = E_d + Q$ , spin-orbit forces can also be important

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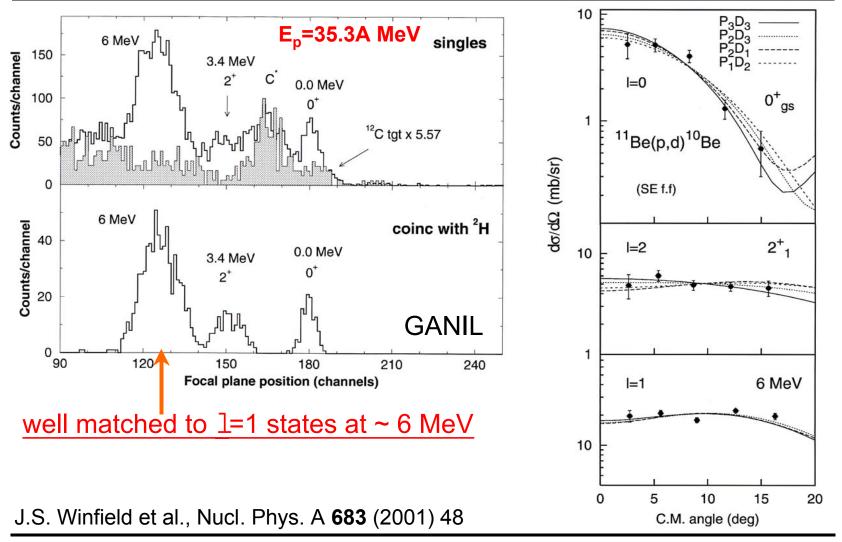
# Medium energy transfer reactions - matching







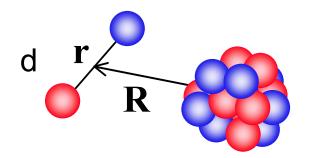
# Angular momentum matching transfer reactions





# Adiabatic model for transfer reactions: e.g. (d,p)

$$T_{dp} = \left\langle \chi_{p}^{(-)}(\mathbf{r}_{p})\phi_{n}(\mathbf{r}_{n}) \middle| V_{np} \middle| \Psi_{K}^{(+)}(\mathbf{r}, \mathbf{R}) \right\rangle \quad \text{note} \quad \mathbf{r} \mid \leq \text{range of } V_{np}$$



$$[T_{\mathbf{R}} + \mathbf{U}(\mathbf{r}, \mathbf{R}) + \mathbf{H}_{\mathbf{d}} - \mathbf{E}] \Psi_{\mathbf{K}}^{(+)} = 0$$

$$H_{d} \rightarrow -\varepsilon_{0}, \ \Psi_{K}^{(+)} \rightarrow \Psi_{K}^{AD}$$
$$[T_{R} + U(\mathbf{r}, \mathbf{R}) - E_{0}] \Psi_{K}^{AD} = 0$$

 $\underline{\mathsf{DWBA}} \ ( | \mathbf{r} | \leq \text{range of } \phi_0 )$ 

$$\Psi_{\mathbf{K}}^{(+)} \rightarrow \phi_{0}(\mathbf{r}) \left\langle \phi_{0}(\mathbf{r}) \middle| \Psi_{\mathbf{K}}^{(+)} \right\rangle_{\mathbf{r}}$$

$$= \phi_{0}(\mathbf{r}) \chi_{\mathbf{K}}^{(+)}(\mathbf{R})$$

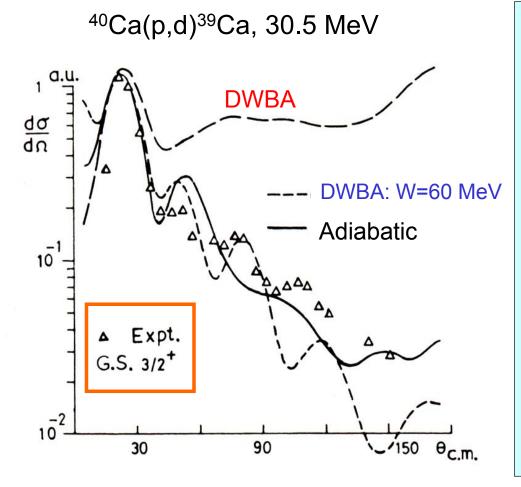
$$= \text{elastic scattering}$$

ADIABATIC ( $|\mathbf{r}| \le \text{range of } V_{np}$ )

$$\begin{aligned} \Psi_{\mathbf{K}}^{\mathrm{AD}} &\approx \phi_{0}(\mathbf{r}) \, \widetilde{\chi}_{\mathbf{K}}^{\mathrm{AD}}(\mathbf{R}) \\ \left[ T_{\mathbf{R}} + \widetilde{\mathbf{V}}(\mathbf{R}) - \mathbf{E}_{0} \right] \, \widetilde{\chi}_{\mathbf{K}}^{\mathrm{AD}} &= 0 \\ \widetilde{\mathbf{V}}(\mathbf{R}) &= \frac{\left\langle \phi_{0} \, | \, \mathbf{V}_{\mathrm{np}} \mathbf{U}(\mathbf{r}, \mathbf{R}) \, | \, \phi_{0} \right\rangle}{\left\langle \phi_{0} \, | \, \mathbf{V}_{\mathrm{np}} \, | \, \phi_{0} \right\rangle} \approx \mathbf{U}(\mathbf{r} = 0, \mathbf{R}) \end{aligned}$$

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# Key features for transfer reactions - spectroscopy



Increased reflection at nuclear surface - less diffuse 'deuteron' channel potential

Greater surface localisation - L-space localisation

Less nuclear volume contribution and less sensitivity to optical model parameters

More consistent sets of deduced spectroscopic factors

J.D. Harvey and R.C. Johnson, Phys. Rev.C 3 (1971) 636



# Wisdom on the choice of distorting potentials

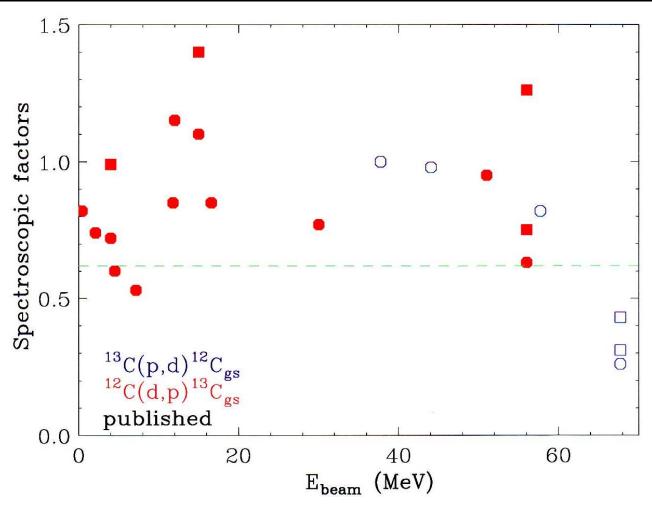
The DWBA analysis of single-nucleon transfer is a semi empirical procedure which describes gross dependences of the cross sections on energy, the Coulomb field, and the atomic weight of the target. This still leaves the novice experimental physicist with the task of choosing the imposing array of parameters needed in such a calculation. It used to be thought that the best procedure is to measure the elastic scattering by the target nucleus of the incident projectiles and that by the final nucleus of the outgoing particles, all at the proper energies, and then to fit the elastic data as well as possible with optical model potentials. These potentials were then to be used as input to DWBA calculations.

Experience has shown that a more sensible procedure is to use distorting parameters which are appropriate for a wider range of target nuclei and energies. Emphasis on accurate fitting of data on one or two nuclei tends to optimize the fit by selecting a peculiar (and perhaps unphysical) set of parameters. In any case, the basic purpose of an optical potential is to describe the average interaction between a projectile and target, and if this interaction turns out to be sharply dependent on the precise energy or target, then the approximations made in assuming an average potential in the first place are likely to be wrong.

M.H. Macfarlane and J.P. Schiffer, Nucl. Spectroscopy and Reactions, Vol B, pp 169



# Spectroscopic factors from individual analyses



X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), in preparation



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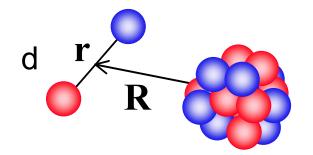
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M.H. Macfarlane and J.P. Schiffer, Nucl. Spectroscopy and Reactions, Vol B, pp 169



# Adiabatic model for transfer reactions: e.g. (d,p)

$$T_{dp} = \left\langle \chi_{p}^{(-)}(\mathbf{r}_{p})\phi_{n}(\mathbf{r}_{n}) \middle| V_{np} \middle| \Psi_{K}^{(+)}(\mathbf{r}, \mathbf{R}) \right\rangle \quad \text{note} \quad \mathbf{r} \mid \leq \text{range of } V_{np}$$



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$$H_{d} \rightarrow -\varepsilon_{0}, \ \Psi_{K}^{(+)} \rightarrow \Psi_{K}^{AD}$$
$$[T_{R} + U(\mathbf{r}, \mathbf{R}) - E_{0}] \Psi_{K}^{AD} = 0$$

 $\underline{\mathsf{DWBA}} \ ( | \mathbf{r} | \leq \text{range of } \phi_0 )$ 

$$\Psi_{\mathbf{K}}^{(+)} \rightarrow \phi_{0}(\mathbf{r}) \left\langle \phi_{0}(\mathbf{r}) \middle| \Psi_{\mathbf{K}}^{(+)} \right\rangle_{\mathbf{r}}$$

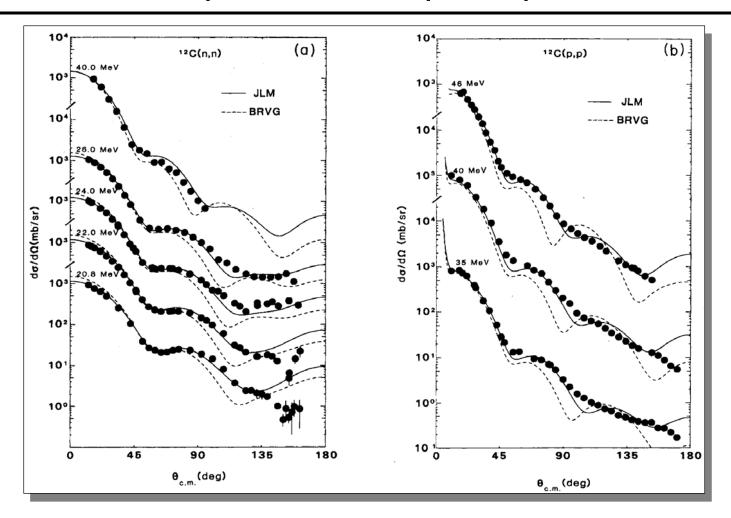
$$= \phi_{0}(\mathbf{r}) \chi_{\mathbf{K}}^{(+)}(\mathbf{R})$$
elastic scattering

ADIABATIC ( $|\mathbf{r}| \le \text{range of } V_{np}$ )

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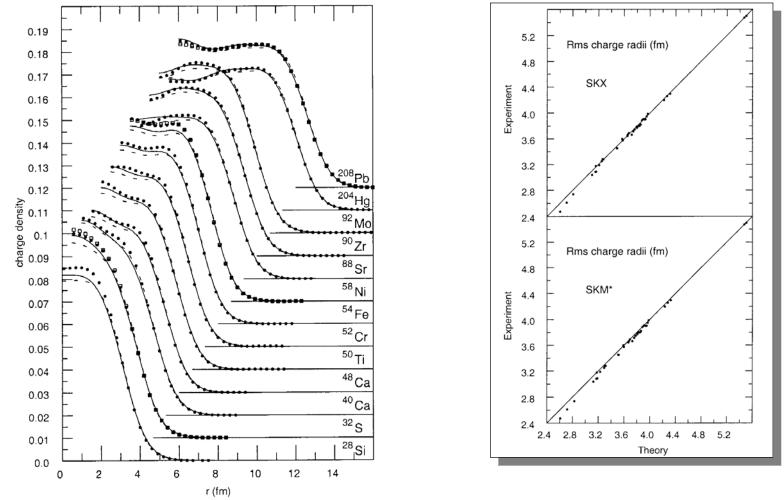
# JLM microscopic nucleon optical potentials



J.S. Petler et al. Phys. Rev. C 32 (1985), 673



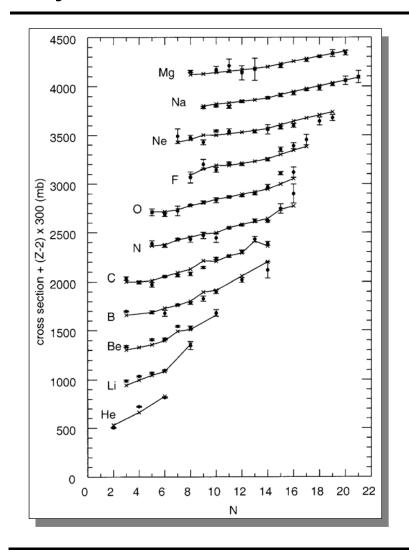
# Skyrme Hartree-Fock radii and densities (2)

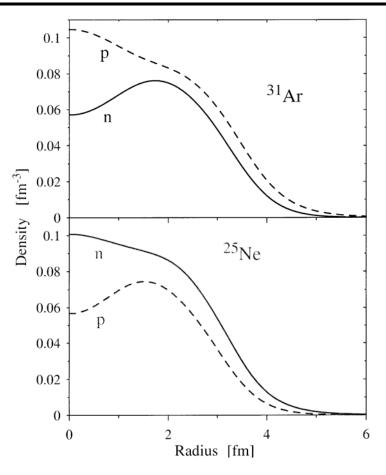


W.A. Richter and B.A. Brown, Phys. Rev. C67 (2003) 034317



# Skyrme Hartree-Fock radii and densities (1)

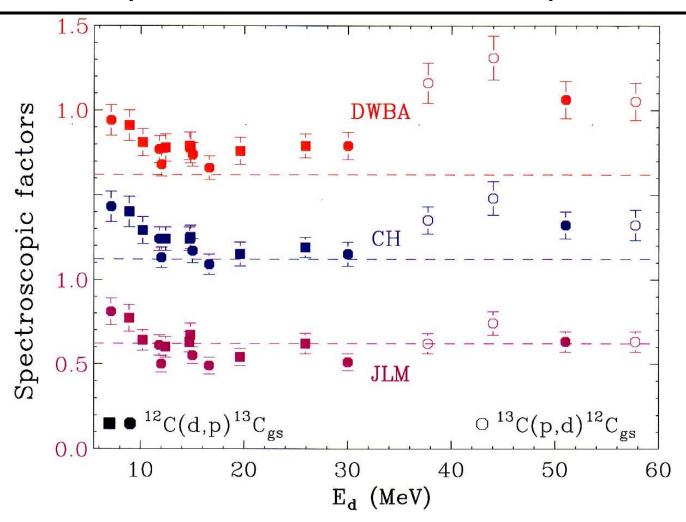




B.A. Brown, S. Typel, and W.A. Richter, Phys. Rev. **C65** (2002) 014612



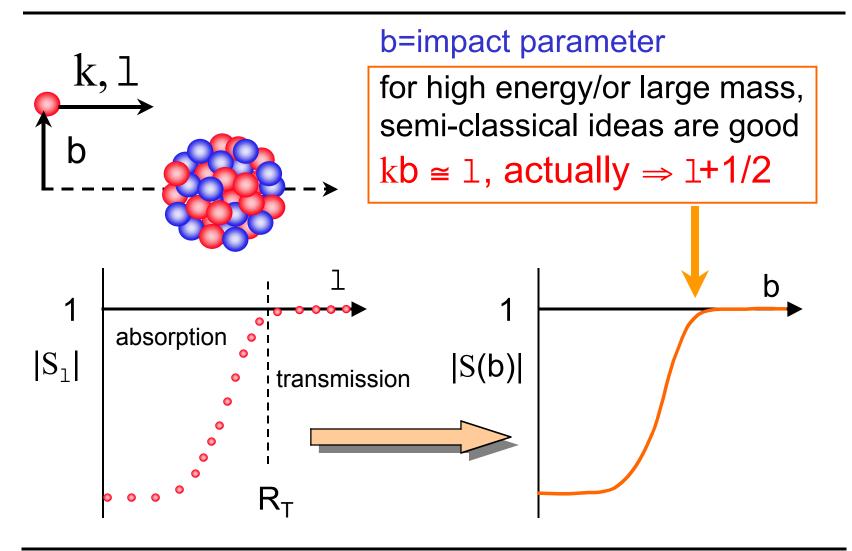
#### Spectroscopic factors – consistent inputs



X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), in preparation



#### The semi-classical methods



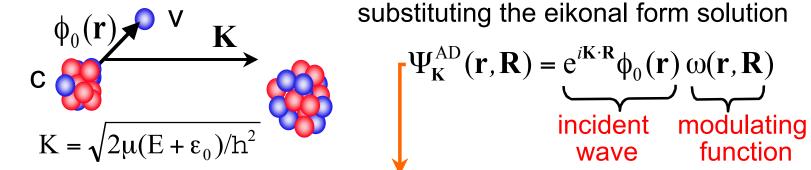


#### Eikonal solution of the few-body model

Practical application of adiabatic approximation:  $H_p \rightarrow -\epsilon_0$ 

$$H = T_{\mathbf{R}} + U(\mathbf{r}, \mathbf{R}) + H_{\mathbf{p}}$$

$$H^{AD} = T_{\mathbf{R}} + U(\mathbf{r}, \mathbf{R}) - \varepsilon_{0}$$



$$[T_{\mathbf{R}} + U(\mathbf{r}, \mathbf{R}) - (E + \varepsilon_0)] \Psi_{\mathbf{K}}^{AD}(\mathbf{r}, \mathbf{R}) = 0$$

and neglecting the curvature term  $\nabla_{\mathbf{R}}^2 \omega(\mathbf{r}, \mathbf{R}) << 2\nabla_{\mathbf{R}} \omega \cdot \mathbf{K}$ 

$$\omega(\mathbf{r}, \mathbf{R}) = \exp \left\{ -\frac{i}{\text{hv}} \int_{-\infty}^{Z} dZ' \left( \mathbf{U}(\mathbf{r}, \mathbf{R}') \right) \right\} \qquad V_{cT} + V_{vT}$$



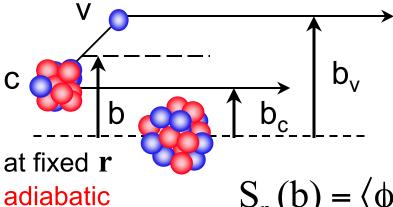
#### Few-body eikonal model amplitudes

So, after the collision, as  $Z \rightarrow \infty$   $\omega(\mathbf{r}, \mathbf{R}) = S_c(b_c) S_v(b_v)$ 

$$\omega(\mathbf{r}, \mathbf{R}) = S_c(b_c) S_v(b_v)$$

$$\Psi_{\mathbf{K}}^{\mathrm{Eik}}(\mathbf{r},\mathbf{R}) \rightarrow \mathrm{e}^{i\mathbf{K}\cdot\mathbf{R}} \, \mathrm{S}_{\mathrm{c}}(\mathrm{b}_{\mathrm{c}}) \, \mathrm{S}_{\mathrm{v}}(\mathrm{b}_{\mathrm{v}}) \, \mathrm{\phi}_{\mathrm{0}}(\mathbf{r})$$

with  $S_c$  and  $S_v$  the eikonal approximations to the S-matrices for the independent scattering of c and v from the target - the dynamics



So, elastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state  $\phi_0(\mathbf{r})$  is

$$S_{p}(b) = \langle \phi_{0} | \underbrace{S_{c}(b_{c})}_{c} \underbrace{S_{v}(b_{v})}_{c} | \phi_{0} \rangle_{r}$$

averaged over position probabilities of c and v

amplitude that c,v survive interaction with b<sub>c</sub> and b<sub>v</sub>



# Dynamics and structure - formal transparency

Independent scattering information of c and v from target

$$S_{\alpha\beta}(b) = \langle \phi_{\beta} \mid S_{c}(b_{c}) S_{v}(b_{v}) \mid \phi_{\alpha} \rangle$$
structure

Use the best available few- or many-body wave functions

More generally,

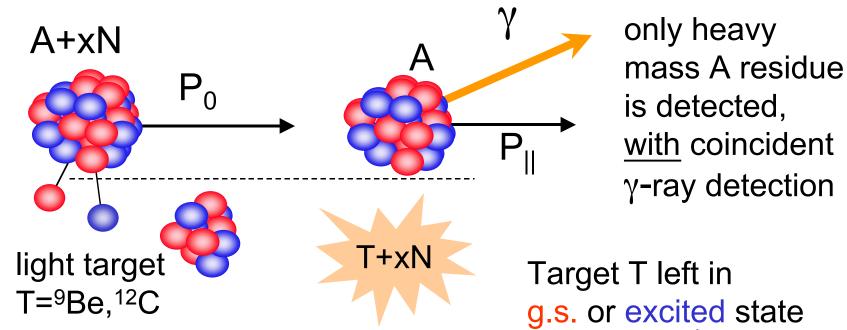
$$S_{\alpha\beta}(b) = \langle \varphi_{\beta} \mid S_1(b_1) S_2(b_2) \dots S_n(b_n) \mid \varphi_{\alpha} \rangle$$

for any choice of 1,2,3, ..... n clusters for which a realistic wave function  $\varphi$  is available



#### One- and two-nucleon knockout reactions

Peripheral collisions (E ≥ 50A MeV; MSU, RIKEN, GSI)



Direct from the projectile perspective





Events contributing will be both <u>break-up</u> and <u>stripping</u> both of which leave a mass A residue in the final state



# Absorptive cross sections - target excitation

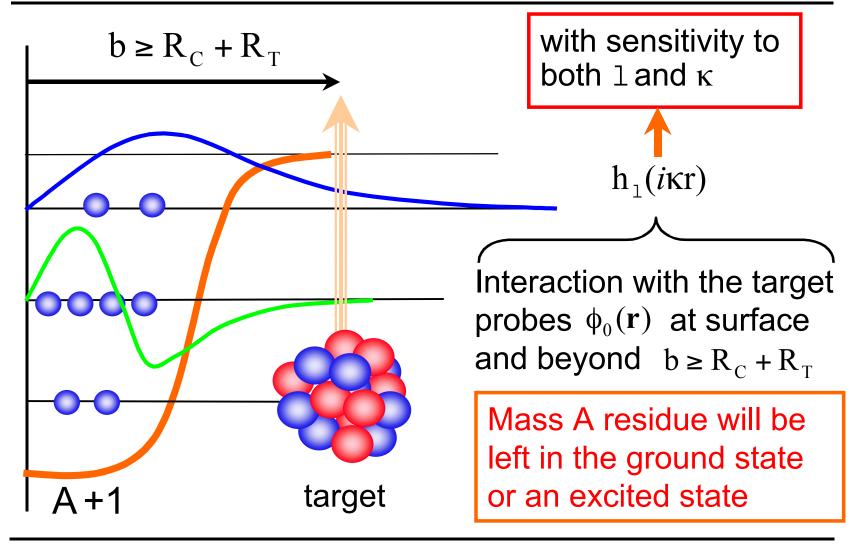
Since our effective interactions are complex all our S(b) include the effects of absorption due to inelastic channels

$$\longrightarrow |S(b)|^2 \le 1$$

Related equations exist for the differential cross sections, etc.

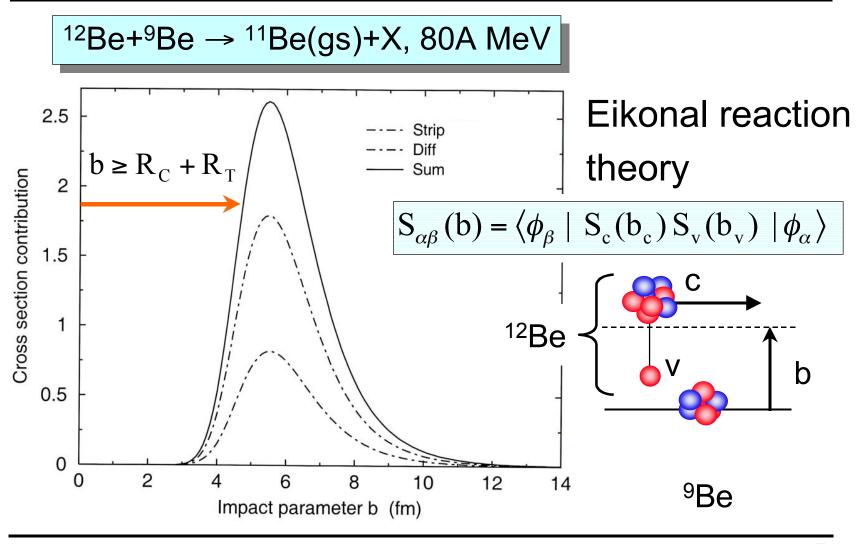


## Probing the surface and tails of wave functions



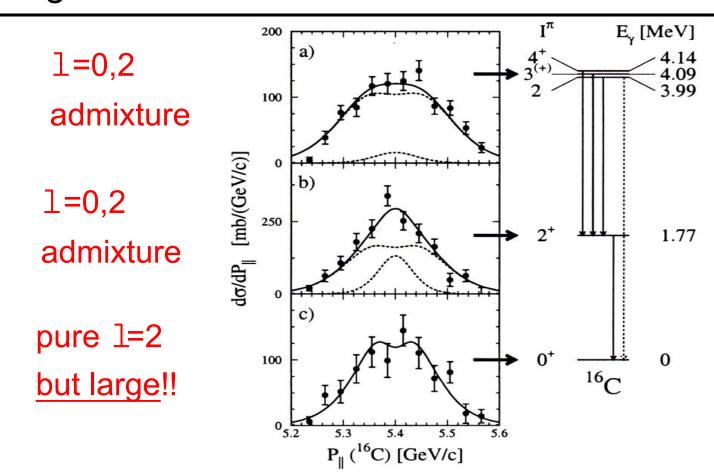


# Contributions are from surface and beyond





# Single-neutron knockout from <sup>17</sup>C - eikonal



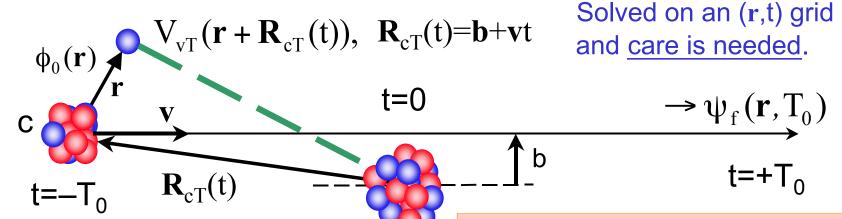
$$\sigma(nI^{\pi}) = \sum_{n} C^{2}S(j, nI^{\pi}) \sigma_{sp}(j, B_{n})$$

V. Maddalena et al. Phys. Rev. C **63** (2001) 024613



#### Non-adiabatic - but trajectory based

Time-dependent (finite difference) solution of the valence particle motion - assuming the heavy core, or c.m., follows a trajectory: [See: Bertsch and Esbensen, Baur and Typel, Suzuki, Melezhik and Baye]



Not exact - but non-adiabatic Dynamics of  $V_{cT}$  is not included and no energy transfer/sharing between core and internal motion. For heavy targets - Coulomb path

$$ih \frac{\partial \psi}{\partial t} = (H_p + V_{vT}) \psi(\mathbf{r}, t)$$
as  $t \to -\infty$   $\psi(\mathbf{r}, t) \to \phi_0(\mathbf{r})$ 

$$t \to +\infty$$
  $\psi(\mathbf{r}, t) \to \psi_f(\mathbf{r}, T_0)$ 



## The time-dependent approach - observables

$$ih \frac{\partial \psi}{\partial t} = (H_p + V_{vT}) \psi(\mathbf{r}, t)$$
as  $t \to -\infty$   $\psi(\mathbf{r}, t) \to \phi_0(\mathbf{r})$ 

$$t \to +\infty$$
  $\psi(\mathbf{r}, t) \to \psi_f(\mathbf{r}, T_0)$ 

absorptive effects of target have to be put in 'by hand' - restricting impact parameters b to values  $b > b_{min} \approx R_T + R_c$ 

Only absorption/loss of flux in the equation is due to  $V_{\nu T}$  and so

At an impact parameter b then (for a neutron valence particle):

neutron removal probability neutron stripping probability diffractive break-up probability

with cross sections

$$P_{-n}(b) = 1 - |\langle \phi_0 | \psi_f \rangle|^2$$

$$P_{str}(b) = 1 - \langle \psi_f | \psi_f \rangle$$

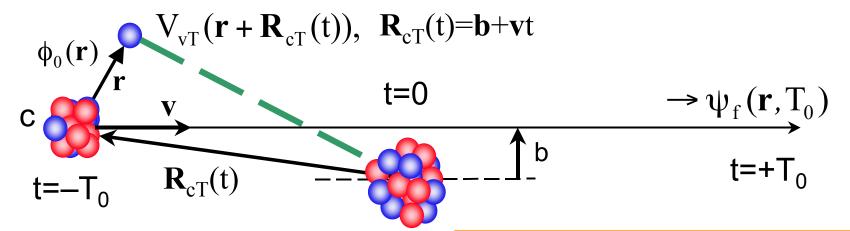
$$P_{\text{diff}}(b) = \langle \psi_f | \psi_f \rangle - |\langle \phi_0 | \psi_f \rangle|^2$$

$$\sigma_{\alpha} = 2\pi \int_{b_{min}} db \ b \ P_{\alpha}(b)$$

**UniS** 

# Transfer to the continuum approximation

Related *transfer to the continuum model* is due to Angela Bonaccorso and David Brink (later in meeting). By use of additional approximations (asymptotic forms of wave function) the finite difference solution is avoided in favour of largely analytic approach.

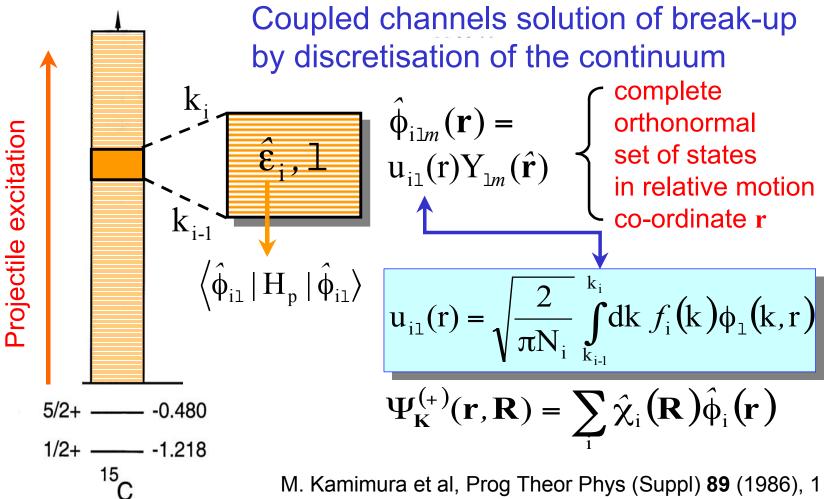


Not exact - but non-adiabatic Dynamics of  $V_{cT}$  is not included and no energy transfer/sharing between core and internal motion.

$$ih\frac{\partial \psi}{\partial t} = (H_p + V_{vT})\psi(\mathbf{r}, t)$$
as  $t \to -\infty$   $\psi(\mathbf{r}, t) \to \phi_0(\mathbf{r})$ 



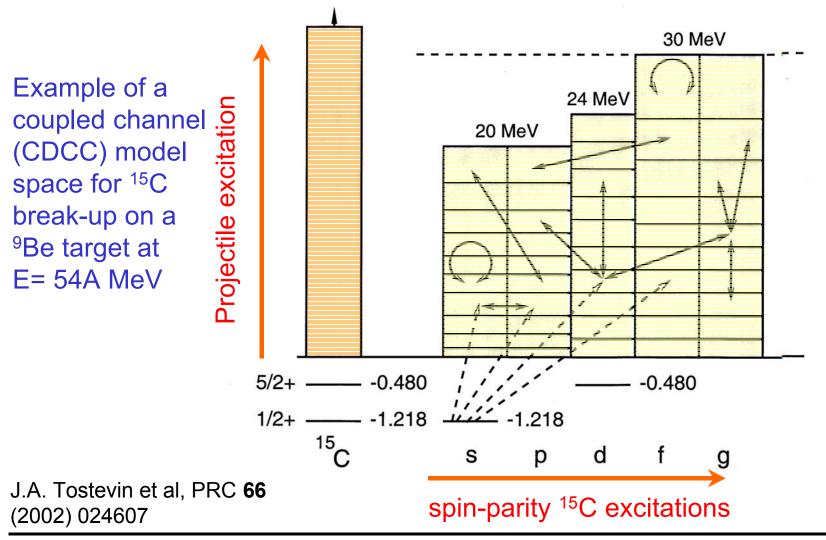
#### Beyond the adiabatic limit - the CDCC



M. Kamimura et al, Prog Theor Phys (Suppl) **89** (1986), 7 N.Austern et al., Phys. Rep. **154** (1987), 125

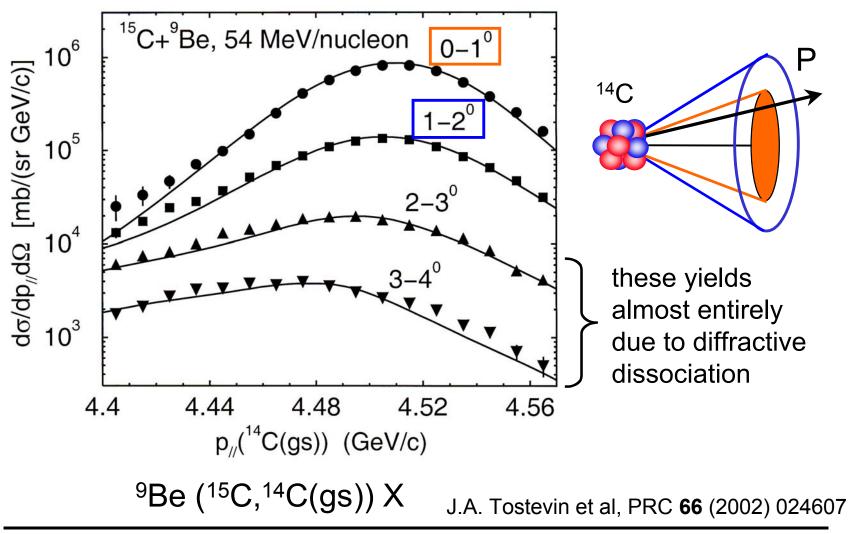


## Coupled channels model space is needed



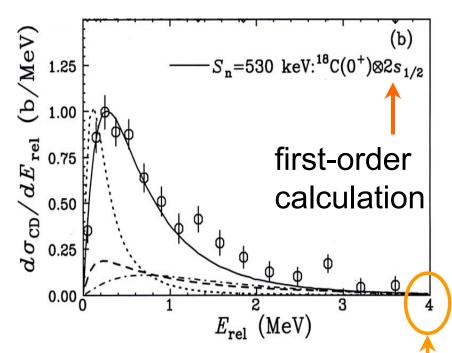


# Core fragment differential cross sections





#### Coupled channels and Coulomb break-up



T. Nakamura et al, PRL 83 (1998) 1112

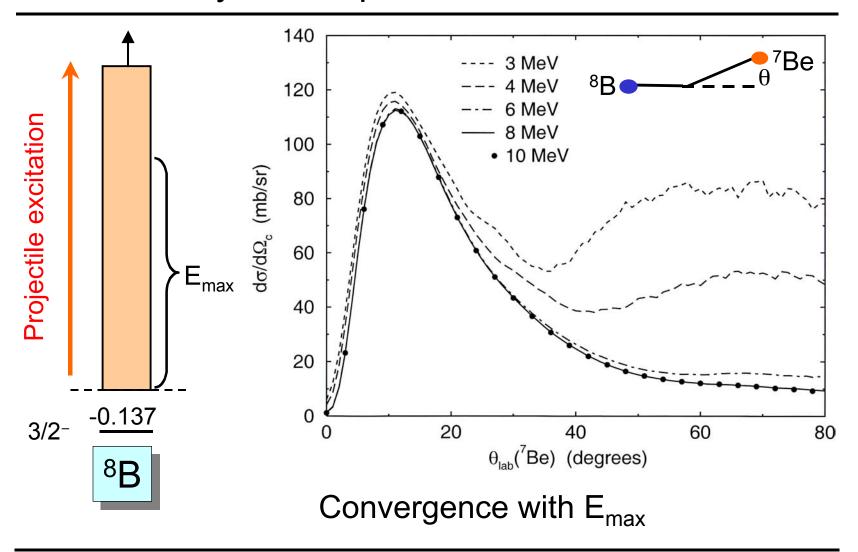
Do CDCC calculations converge in the case of Coulomb couplings?

 $\Delta k = k_i - k_{i-1}$  must be small

 $\hat{\phi}_{i}(\mathbf{r})$  and associated couplings  $\langle \hat{\phi}_{i} | U(\mathbf{r}, \mathbf{R}) | \hat{\phi}_{j} \rangle$  of very long range

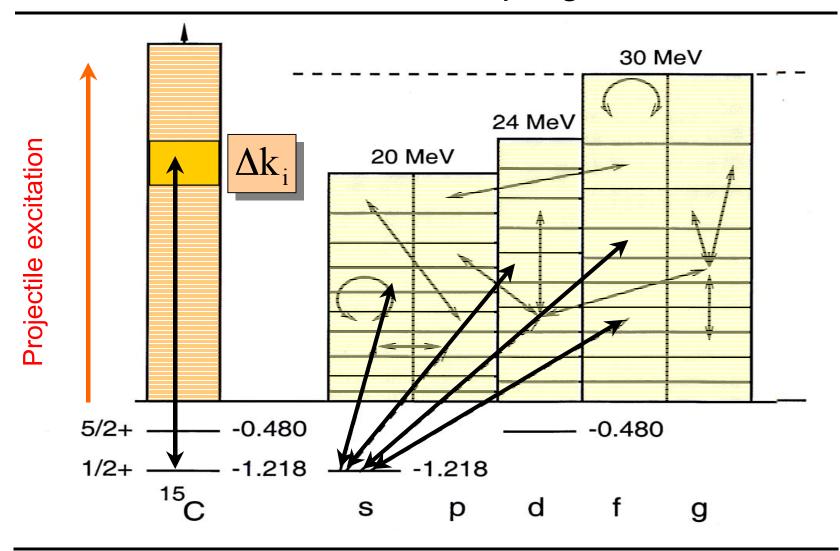


# <sup>8</sup>B - a weakly bound proton nucleus



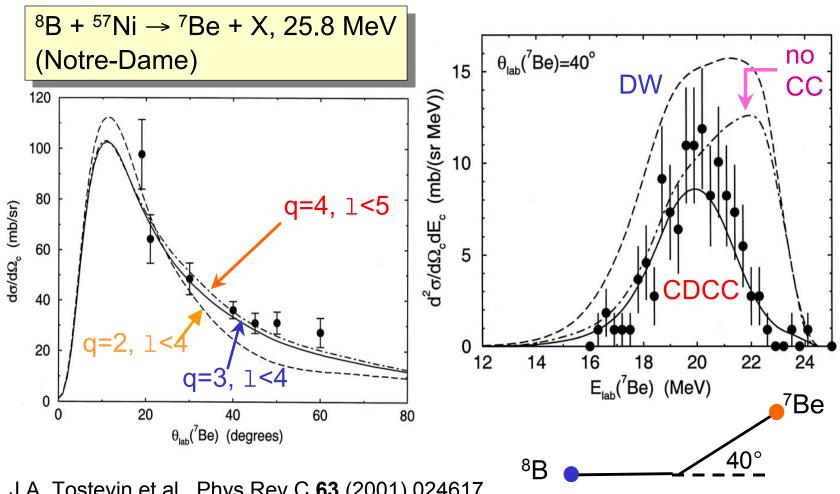


# No continuum-continuum couplings





# CDCC can reproduce data at low energy



J.A. Tostevin et al., Phys Rev C **63** (2001) 024617

J. Kolata et al., Phys Rev C **63** (2001) 024616



# Double differential cross sections for breakup

